[Chung's Equations Models_Theories]

▣ **Chung's equations 1, 2,** and **3** were established to predict and evaluate the risk of smoke during a fire [Appl. Chem. Eng., Vol. 29, No. 6, 2018, 670-676].

* **Smoke Performance Index (A, B), SPI (A, B) :**

$$\boldsymbol{SPI\ (s^2/m^2)\ (A)} = \frac{\boldsymbol{TTI\ (s)}}{\boldsymbol{SPR_{peak}(m^2/s)}} \quad (1)$$

TTI : Time to ignition, SPR_{peak}: $1_{st\text{-}peak}$ smoke production rate

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

* **Smoke Growth Index (A, B), SGI (A, B) :**

$$\boldsymbol{SGI\ (m^2/s_2)\ (A)} = \frac{\boldsymbol{SPR_{peak}\ (m^2/s)}}{\boldsymbol{Time\ to\ SPR_{peak}(s)}} \quad (2)$$

Time to SPR_{peak} : Time to $1_{st\text{-}peak}$ smoke production emission

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

* **Smoke Intensity, SI :**

$$\boldsymbol{SI\ (kW/kg) = MARHE\ (kW/m^2)\ \cdot\ SEA\ (m^2/kg)} \quad (3)$$

MARHE : Maximum average rate of heat emission

SEA : Specific extinction area

.

▣ To standardize the smoke risk assessment, **Chung's equations-V** [Smoke Performance Index-V (A, B), SPI-V (A, B)] and [Smoke Growth Index-V (A, B), SGI-V (A, B)] were established using reference materials [Appl. Chem. Eng., Vol. 32, No. 2, 2021, 197-204].

* **Smoke Performance Index-V (A, B), SPI-V (A, B) :** Dimensionless index

$$SPI-V\,(A) = \frac{\dfrac{TTI\,(s)}{SPR_{peak}\,(m^2/s)}}{\left[\dfrac{TTI\,(s)}{SPR_{peak}\,(m^2/s)}\right]_{PMMA}} \qquad (4)$$

∗ Method B : Replace SPR_{peak} with $SEA\ (m^2/kg)$.

PMMA : Polymethyl methacrylate

* **Smoke Growth Index-V (A, B), SGI-V (A, B) :** Dimensionless index

$$SGI-V\,(A) = \frac{\dfrac{SPR_{peak}\,(m^2/s)}{Time\ to\ SPR_{peak}\,(s)}}{\left[\dfrac{SPR_{peak}\,(m^2/s)}{Time\ to\ SPR_{peak}\,(s)}\right]_{PMMA}} \qquad (5)$$

∗ Method B : Replace $SPR_{peak}(m^2/s)$ with $SEA\ (m^2/kg)$.

PMMA : Polymethyl methacrylate

▣ In addition, to predict and evaluate overall smoke risk, **Chung's equation-VI** was established based on the relationship between the Smoke Performance Index-V (A, B), SPI-V (A, B) and the Smoke Growth Index-V (A, B), SGI-V (A, B) [Appl. Chem. Eng., Vol. 32, No. 2, 2021, 197-204].

* **Smoke Risk Index-VI (A, B), SRI-VI (A, B)] :** Dimensionless index

$$SRI-VI\,(A) = \frac{SGI-V\,(A)}{SPI-V\,(A)} \qquad SRI-VI\,(B) = \frac{SGI-V\,(B)}{SPI-V\,(B)} \qquad (6)$$

.

▣ Since thermal characteristics exhibit stronger values than smoke characteristics during a fire, the fire risk was implemented more quantitatively by reflecting thermal characteristics in the smoke characteristics. Accordingly, **Chung's equations-II** [Fire Performance Index-II (A, B), FPI-II (A, B)] and [Fire Growth Index-II (A, B), FGI-II (A, B)] were established as follows [Appl. Chem. Eng., Vol. 31, No. 3, 2021, 277-283].

* **Fire Performance Index-II (A, B), FPI-II (A, B) :**

$$\boldsymbol{FPI-II\ (s^2/kW)(A) = \frac{TTI\ (s)}{SPR_{peak}(m^2/s) \cdot PHRR\ (kW/m^2)}} \tag{7}$$

PHRR : $1_{st\text{-}peak}$ heat release rate

MARHE : Maximum Average Rate of Heat Emission

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR\ (kW/m^2)$ with $MARHE$ (kW/m^2).

* **Fire Growth Index-II (A, B), FGI-II (A, B) :**

$$\boldsymbol{FGI-II\ (kW/s^2)(A) = \frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}{Time\ to\ SPR\ (s)}} \tag{8}$$

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR\ (kW/m^2)$ with $MARHE$ (kW/m^2).

▣ In addition, to standardize the fire risk assessment, **Chung's equations-III** [Fire Performance Index-III (A, B), FPI-III (A, B)] and [Fire Growth Index-III (A, B), FGI-III (A, B)] were established using reference materials [Appl. Chem. Eng., Vol. 32, No. 1, 2021, 75-82].

* **Fire Performance Index-III (A, B), FPI-III (A, B)] :** Dimensionless index

$$\boldsymbol{FPI-III\ (A) = \frac{\frac{TTI\ (s)}{SPR_{peak}(m^2/s) \cdot PHRR\ (kW/m^2)}}{\left[\frac{TTI\ (s)}{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}\right]_{PMMA}}} \tag{9}$$

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR\ (kW/m^2)$ with $MARHE$ (kW/m^2).

PMMA : Polymethyl methacrylate

* **Fire Growth Index-III (A, B), FGI-III (A, B)]** : Dimensionless index

$$FGI-III\ (A) = \frac{\frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}{Time\ to\ SPR_{peak}\ (s)}}{\left[\frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}{Time\ to\ SPR_{peak}\ (s)}\right]_{PMMA}} \qquad (10)$$

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR\ (kW/m^2)$ with $MARHE$ (kW/m^2).

PMMA : Polymethyl methacrylate

▣ To conduct a comprehensive fire risk assessment, **Chung's equation-IV** was established based on the relationship between [Fire Performance Index-III (A, B), FPI-III (A, B)] and [Fire Growth Index-III (A, B), FGI-III (A, B)] [Appl. Chem. Eng., Vol. 32, No. 1, 2021, 76-82].

* **Fire Risk Index-IV (A, B), FRI-IV (A, B)]** : Dimensionless index

$$FRI-IV\ (A) = \frac{FGI-III\ (A)}{FPI-III\ (A)} \qquad FRI-IV\ (B) = \frac{FGI-III\ (B)}{FPI-III\ (B)} \qquad (11)$$

▣ **Chung's equations-VII** [Fire Performance Index-VII (A, B), FPI-VII (A, B)] and [Fire Growth Index-VII (A, B), FGI-VII (A, B)], including the average generation rate ratios of carbon monoxide and carbon dioxide which were treated separately, were established [Appl. Chem. Eng. Vol. 34, No. 2, 2023, 144-152]

* **Fire Performace Index-VII (A, B), FPI-VII (A, B)] :**

$$FPI-VII\ (s^2/\mathrm{kW})\ (A) = \frac{TTI\ (s)}{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s)/CO_2P_{mean}\ (g/s)} \qquad (12)$$

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR\ (kW/m^2)$ with $MARHE$ (kW/m^2).

.

* **Fire Growth Index-VII (A, B), FGI-VII (A, B)] :**

$$FGI-VII\ (kW/s^2)\ (A) = \frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s)\ /\ CO_2P_{mean}\ (g/s)}{Time\ to\ SPR_{peak}\ (s)} \quad (13)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

▣ To standardize fire risk, **Chung's equations-VIII** [Fire Performance Index-VIII (A, B), FPI-VIII (A, B)] and [Fire Growth Index-VIII (A, B), FGI-VIII (A, B)] were established. Here, FPI-VIII is defined as the value obtained by dividing FPI-VII by FPI-VII$_{[PMMA]}$ (based on PMMA test specimens) [Appl. Chem. Eng., Vol. 34, No. 2, 2023, 144-152].

* **Fire Performance Index-VIII (A, B), FPI-VIII (A, B)** : Dimensionless index

$$FPI-VIII\ (A) = \frac{\frac{TTI\ (s)}{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)\ \cdot COP_{mean}\ (g/s)\ /\ CO_2P_{mean}\ (g/s)}}{\left[\frac{TTI\ (s)}{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2 \cdot COP_{mean}(g/s)\ /\ CO_2P_{mean}(g/s)}\right]_{PMMA}} \quad (14)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

* **Fire Growth Index-VIII (A, B), FGI-VIII (A, B)** : Dimensionless index

$$FGI-VIII\ (A) = \frac{\frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s)\ /\ CO_2P_{mean}\ (g/s)}{Time\ to\ SPR_{peak}}}{\left[\frac{SPR_{peak}(m^2/s) \cdot PHRR(kW/m^2) \cdot COP_{mean}(g/s)\ /\ CO_2P_{mean}(g/s)}{Time\ to\ SPR_{peak}}\right]_{PMMA}} \quad (15)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

Here, FGI-VIII is defined as the value obtained by dividing FGI-VII by FGI-VII $_{[PMMA]}$ (based on PMMA test specimens).

▣ This is the same as the equation (16) expressing the fire risk and is consistent with the comprehensive fire risk formula for evaluating fire risk.

$$\boldsymbol{Fire\ Risk} \propto \frac{\boldsymbol{FGI-VIII}}{\boldsymbol{FPI-VIII}} \text{ or } \frac{\boldsymbol{FGI-III}}{\boldsymbol{FPI-III}} \text{ or } \frac{\boldsymbol{FGI-II}}{\boldsymbol{FPI-II}} \text{ or } \frac{\boldsymbol{SGI-XI}}{\boldsymbol{SPI-XI}} \tag{16}$$

▣ **Chung's equation-IX** [(Fire Risk Index-IX (A, B), FRI-IX (A, B)] was established by improving the Fire Risk Index-IV (A, B) [Fire Risk Index-IV (A, B), FRI-IV (A, B)] to comprehensively predict fire risk and fire risk grades. FRI-IX (A, B) is defined as the value obtained by dividing FGI-VIII (A, B) by FPI-VIII (A, B) [Appl. Chem. Eng., Vol. 34, No. 2, 2023, 144-152].

∗ **Fire Risk Index-IX (A, B), FRI-IX (A, B)]** : Dimensionless index

$$\boldsymbol{FRI-IX\ (A)} = \frac{\boldsymbol{FGI-VIII\ (A)}}{\boldsymbol{FPI-VIII\ (A)}} \qquad \boldsymbol{FRI-IX\ (B)} = \frac{\boldsymbol{FGI-VIII\ (B)}}{\boldsymbol{FPI-VIII\ (B)}} \tag{17}$$

Table 1. Criteria of Fire Risk Rating for the Fire Risk Index-IX (A)

FRI-IX (A)	Fire risk rating	Fire safety
5 or less	A	Very high
More than 5 up to 10	B	High
More than 10 up to 15	C	Medium 1
More than 15 up to 20	D	Medium 2
More than 20 up to 25	E	Low 1
More than 25 up to 30	F	Low 2
Over 30	G	Very low

▣ In another way, **Chung's equations-X** were newly established by introducing the Combustion Resistance Time (CRT,18) as a replacement element for the ignition time (TTI) of the combustible material, and the Accumulated Smoke Generation Time (ASGT, 19) as a replacement element for the time to reach the maximum smoke generation velocity of the combustible material [Time to SPR_{1st_peak} (s), $TSPR_{1st_peak}$ (s)] [Appl. Chem. Eng., Vol. 34, No. 4, 2023, 388-396].

∗ **Fire Performance Index-X (A, B), FPI-X (A, B)] :**

$$\boldsymbol{FPI-X\ (s^2/kW)\ (A)} = \frac{\boldsymbol{CRT\ (s)}}{\boldsymbol{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s)/CO_2P_{mean}\ (g/s)}} \tag{18}$$

CRT : Combution Resistance Time

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

∗ **Fire Performance Index-XIII (A, B), FPI-XIII (A, B) :** [Appl. Chem. Eng., Vol. 37, No. 1, 2026, 81-87].

$$\boldsymbol{FPI-XIII\ (s^2/kW)\ (A)} = \frac{\boldsymbol{CRT\ (s)}}{\boldsymbol{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}} \quad (19)$$

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

▣ To standardize the fire risk, **Chung's equations-XI** [Fire Performance Index-XI (A, B), FPI-XI (A, B)] and [Fire Growth Index-XI (A, B), FGI-XI (A, B)] were established [Appl. Chem. Eng., Vol. 34, No. 4, 2023, 388-396].

∗ **Fire Performance Index-XI (A, B), FPI-XI (A, B) :** Dimensionless index

FPI-XI is expressed as the value obtained by dividing FPI-X by the PMMA reference value, FPI-X [PMMA].

$$\boldsymbol{FPI-XI\ (A)} = \frac{\dfrac{\boldsymbol{CRT\ (s)}}{\boldsymbol{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s) / CO_2P_{mean}\ (g/s)}}}{\left[\dfrac{\boldsymbol{CRT\ (s)}}{\boldsymbol{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}(g/s) / CO_2P_{mean}(g/s)}}\right]_{PMMA}} \quad (20)$$

∗ Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

.

* **Fire Performance Index-XIV (A, B), FPI-XIV (A, B)] :** Dimensionless index [Appl. Chem. Eng., Vol. 37, No. 1, 2026, 81-87].

$$FPI-XIV\ (A) = \frac{\dfrac{CRT\ (s)}{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}}{\left[\dfrac{CRT\ (s)}{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}\right]_{PMMA}} \quad (21)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

* **Fire Growth Index-X (A, B), FGI-X (A, B)] :**

$$FGI-X\ (kW/s^2)\ (A) = \frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s)/CO_2P_{mean}(g/s)}{ASGT\ (s)} \quad (22)$$

ASGT : Accumulated Smoke Generation Time

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

* **Fire Growth Index-XIII (A, B), FGI-XIII (A, B)] :** [Appl. Chem. Eng., Vol. 37, No. 1, 2026, 81-87].

$$FGI-XIII\ (kW/s^2)\ (A) = \frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2)}{ASGT\ (s)} \quad (23)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

.

* **Fire Growth Index-XI (A, B), FGI-XI (A, B)]** : Dimensionless index

FGI-XI (A, B) is expressed as the value obtained by dividing FGI-X (A, B) by the PMMA reference value FGI-X [PMMA].

$$FGI-XI\ (A) = \frac{\frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s) / CO_2P_{mean}\ (g/s)}{ASGT\ (s)}}{\left[\frac{SPR_{peak}\ (m^2/s) \cdot PHRR\ (kW/m^2) \cdot COP_{mean}\ (g/s) / CO_2P_{mean}\ (g/s)}{ASGT\ (s)}\right]_{PMMA}} \quad (24)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

* **Fire Growth Index-XIV (A, B), FGI-XIV (A, B)] :** Dimensionless index [Appl. Chem. Eng., Vol. 37, No. 1, 2026, 81-87].

$$FGI-XIV\ (A) = \frac{\frac{SPR_{peak}(m^2/s) \cdot PHRR(kW/m^2)}{ASGT\ (s)}}{\left[\frac{SPR_{peak}(m^2/s) \cdot PHRR(kW/m^2)}{ASGT\ (s)}\right]_{PMMA}} \quad (25)$$

* Method B : Replace SPR_{peak} with SEA (m^2/kg).

Replace $PHRR$ (kW/m^2) with $MARHE$ (kW/m^2).

▣ This equation (26) corresponds to a comprehensive fire risk formula that evaluates the fire risk in relation to each other.

$$Fire\ Risk \propto \frac{FGI-XIV}{FPI-XIV} \text{ or } \frac{FGI-XI}{FPI-XI} \text{ or } \frac{FGI-VIII}{FPI-VIII} \text{ or } \frac{FGI-III}{FPI-III} \text{ or } \frac{FGI-II}{FPI-II} \text{ or } \frac{SGI-XI}{SPI-XI} \quad (26)$$

▣ **Chung's equation-XII** [(Fire Risk Index-XII, FRI-XII)] was established by improving the Fire Risk Index-IV (FRI-IV) to comprehensively predict another fire risk and fire risk grade. FRI-XII (A, B) is defined as the value obtained by dividing FGI-XI (A, B) by FPI-XI (A, B) [Appl. Chem. Eng., Vol. 34, No. 4, 2023, 388-396].

* **Fire Risk Index-XII (A, B), FRI-IX (A, B)] :** Dimensionless index

$$FRI-XII\ (A) = \frac{FGI-XI\ (A)}{FPI-XI\ (A)} \qquad FRI-XII\ (B) = \frac{FGI-XI\ (B)}{FPI-XI\ (B)} \quad (27)$$

* **Fire Risk Index-XV (A, B), FRI-XV (A, B)] :** Dimensionless index [Appl. Chem. Eng., Vol. 37, No. 1, 2026, 81-87].

$$\boldsymbol{FRI-XV\ (A)} = \frac{\boldsymbol{FGI-XIV\ (A)}}{\boldsymbol{FPI-XIV\ (A)}} \qquad \boldsymbol{FRI-XV\ (B)} = \frac{\boldsymbol{FGI-XIV\ (B)}}{\boldsymbol{FPI-XIV\ (B)}} \tag{28}$$

As the FRI-XII (A, B) or FRI-XV (A, B) values increase, the fire risk increases, and conversely, as the FRI-XII (A, B) or FRI-XV (A, B) values decrease, the fire risk and the fire risk grade can be comprehensively predicted.

Table 2. New Criteria for Fire Risk Rating for Fire Risk Index-XII (A) [Appl. Chem. Eng., Vol. 36, No. 2, 2025, 216-223].

FRI-XII (A)	Fire Risk Rating	Fire Safety
1 or less	A	Very high
More than 1 up to 1.5	B	High
More than 1.5 up to 2.0	C	Medium 1
More than 2.0 up to 2.5	D	Medium 2
More than 2.5 up to 3.0	E	Low 1
More than 3.0 up to 3.5	F	Low 2
Over 3.5	G	Very low